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6 Control Law for a Missile Intercept System.

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Abstract

A feedback control law is derived for a two dimensional missile intercept system which combines in a step-by-step manner the closed form solutions to a least squares estimation of the target turn rate, line speed and heading with a minimum control energy problem subject to a terminal intercept constraint. The major assumptions are that the pursuer possesses thrust modulation capabilities, in addition to thrust vectoring, and that continuous measurements of the range coordinates are available for estimation purposes. Simulation results are included which indicate that good terminal accuracy is achieved when the estimation errors are small and illustrate the limitations on accuracy when fluctuations occur in the target speed and turn rate.

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I. Introduction

Among the various alternatives which have been proposed for the proportional navigation guidance law in missile intercept problems, Deyst [1] applied optimal stochastic control theory, Ho [2] and Rajan [3] applied the theory of differential games, and Slater [4] and Nazaroff [5] took the perturbation point of view in applying linear optimal control theory. In this paper a deterministic approach to a two dimensional missile intercept problem is considered under the assumptions that the pursuer possesses thrust modulation capabilities, in addition to the usual thrust vectoring, and that continuous measurements of the relative range coordinates are available. The thrust modulation capability facilitates the formulation of the control problem as a commutative bilinear system allowing for the application of some results recently obtained in [6]. The philosophy for deriving the overall control law is basically the same as that used by the authors in [7] for the terminal control of a gliding parachute system in a nonuniform wind, viz. independent and easily computable solutions to the control and estimation problems, relative to a time interval $t_i \leqslant t \leqslant t_{i+1}$, are combined to define a step-by-step control-estimation sequence, $i = 0,1,2\cdots$, which constitutes the closed loop control law.

Following a statement of the problem in Section II, a least squares estimation scheme is developed in Section III to estimate the target speed and relative heading. A closed-form solution to a minimum control energy problem with terminal constraint is obtained in Section IV, assuming the parameters for the target speed and heading are known. A modification is considered in Section V to include actuator dynamics for the thrust vectoring engine. Simulation results are presented in Section VI which combine the estimation parameters from Section III with the controls obtained in IV and V for the closed loop (feedback) control law.

II. Problem Statement

It is assumed for a high speed pursuing missile and short initial range that the maneuvering of the vehicles can be restricted to a two dimensional plane as shown in Fig. 1 with the coordinates fixed in the missile. Denoting the angular rates of the missile and target with respect to a nonrotating reference frame by u_M and u_T respectively, the kinematic equations of motions are decribed by

$$\dot{x}_1 = -v_T \sin x_3 + x_2 u_M$$
$$\dot{x}_2 = v_T \cos x_3 - x_1 u_M - v_M \qquad (1)$$
$$\dot{x}_3 = u_T - u_M$$

where v_T and v_M are the line speeds of the target and missile relative to air, x_1 and x_2 are the horizontal and vertical distances of the target relative to the missile, and x_3 is the relative angle between the headings of the missile and target [5].

With a specified sequence of time instants t_i, i=0,1,2$\cdots$, the estimation problem relative to the $i\underline{th}$ time interval, $t_i \leqslant t \leqslant t_{i+1}$, is to obtain estimates of the parameters $(v_T, u_T, x_3(t_i))$ based on continuous measurements of the position coordinates $(x_1(t), x_2(t))$ and a knowledge of the missile line speed and turning rate (v_M, u_M) on $[t_i, t_{i+1}]$. The control problem relative to the $i\underline{th}$ time instant, t_i, is to obtain a reasonable control strategy for $u_M(t)$ and $v_M(t)$, $t > t_i$, such that an intercept occurs at some future time $T > t_i$, i.e. $x_1(T) = x_2(T) = 0$, assuming that the estimates of the parameters $(v_T, u_T, x_3(t_i))$ are exact from a previous subinterval. This pair of estimation and control problems is solved anew on subsequent time intervals, $t_{i+1} \leqslant t \leqslant t_{i+2}$, etc., in defining the closed loop control law. Precise statements and solutions to these problems are given in the following sections.

III. Least Squares Estimation of (v_T, u_T, x_3)

Let $[t_o, t_1]$ denote a typical time interval over which continuous measurements of the range coordinates $(x_1(t), x_2(t))$ are assumed given together with a knowledge of the missile turn rate and line speed (u_M, v_M). A deterministic least squares estimate of the parameters $(v_T, u_T, x_3(t_o))$ is considered here in order to obtain a simple closed form solution to this aspect of the problem, thereby facilitating an easily implemented control law for the overall missile intercept problem as discussed earlier.

Denote the target speed v_T and initial heading $x_3(t_o)$ by the parameters v and "a", i.e. $(v_T, x_3(t_o)) = (v, a)$, and assume for the time being that the target turn rate u_T is a known constant.[†] A least squares estimate of (a, v) results · upon minimizing the functional:

$$J(a,v) = \int_{t_o}^{t_1} \left\{ \dot{x}_1(t) + v \sin[a+U(t)] - u_M x_2(t) \right\}^2 dt$$

$$+ \int_{t_o}^{t_1} \left\{ \dot{x}_2(t) - v \cos[a+U(t)] + v_M + u_M x_1(t) \right\}^2 dt \tag{2}$$

where $U(t)$ is defined in terms of the target and missile turn rates, u_T and u_M, by:

$$U(t) = \int_{t_o}^{t} [u_T - u_M(s)]ds. \tag{3}$$

A necessary condition for the minimization of J is that the partial derivatives of J with respect to "a" and v vanish:

$$\left.\frac{\partial J}{\partial a}\right|_{(a^*, v^*)} = 0 \qquad , \qquad \left.\frac{\partial J}{\partial v}\right|_{(a^*, v^*)} = 0 \quad . \tag{4}$$

[†] The estimation of u_T will be considered separately at the end of this section.

Observing that J is quadratic in v, the best estimate v* can be uniquely determined in terms of a*:

$$v^* = - \frac{1}{t_1 - t_o} \left\{ x_1(t_1)\sin[a^*+U(t_1)] - x_1(t_o)\sin a^* + x_2(t_o)\cos a^* \right.$$
$$\left. - x_2(t_1)\cos[a^*+U(t_1)] \right\}$$
$$- \int_{t_o}^{t_1} \left\{ u_T \left[x_1(t)\cos[a^*+U(t)] + x_2(t)\sin[a^*+U(t)] \right] + v_M \cos[a^*+U(t)] \right\} dt. \tag{5}$$

Similarly,

$$0 = A \cos a^* + B \sin a^* \tag{6}$$

where

$$A = x_1(t_1) \cos U(t_1) + x_2(t_1) \sin U(t_1) - x_1(t_o)$$
$$+ \int_{t_o}^{t_1} \left\{ [v_M + u_T x_1(t)] \sin U(t) - u_T x_2(t) \cos U(t) \right\} dt$$

$$B = -x_1(t_1) \sin U(t_1) + x_2(t_1) \cos U(t_1) - x_2(t_o)$$
$$+ \int_{t_o}^{t_1} \left\{ [v_M + u_T x_1(t)] \cos U(t) + u_T x_2(t) \sin U(t) \right\} dt.$$

From Equation (6), we obtain

$$a^* = 2m\pi + \tan^{-1} \frac{A}{B} \qquad , \qquad m = 0, \pm 1, \ldots \tag{7}$$

in which m is chosen so that $\partial^2 J/\partial a^2 \Big|_{(a^*,v^*)} > 0$. This is equivalent to:

$$B \cos a^* > 0. \tag{8}$$

Following the solution for a* from (7) and (8), v* is obtained from (5).

The above solution for estimating $(v_T, x_3(t_o))$ assumes a knowledge of u_T. Minimizing (2) over u_T will not lead to a simple closed form solution and, therefore, another approach is necessary to estimate u_T. A new approach to parameter estimation problems has recently been developed in [8] which can be directly applied to this problem. In applying this technique it is first neces- sary to rewrite the underlying differential equations (1) in terms of a dif- ferential operator equation of the generic form

$$0 = P(D)w(t) - Q(D)V(t)f(\theta) \quad , \quad t_o \leqslant t \leqslant t_1 \qquad (9)$$

where $P(D)$ and $Q(D)$ are polynomial matrices in the differential operator $D = \frac{d}{dt}$ given by

$$P(D) = \sum_{i=0}^{n} P_{n-i} D_i \quad , \quad Q(D) = \sum_{i=0}^{n} Q_{n-i} D^i$$

and $(w(t), V(t))$ are vector and matrix valued functions, respectively, of the observed data on $[t_o, t_1]$. The vector valued function $f(\theta)$ in (9) is a given single valued continuously differentiable function of the parameter vector θ which is to be estimated based on the given data over $[t_o, t_1]$. For the missile intercept model (1), the requisite equation can be obtained by differentiating the first equation in (1), substituting from the second and third equations in (1) to eliminate $v_T \cos x_3$ and $\dot{x}_3$, and rearranging the resulting expression to obtain the following differential operator equation:

$$0 = [D^2, D, 1] \begin{bmatrix} -x_1(t) \\ x_2(t)[u_M(t) - 1] \\ u_M(t)[x_1(t)u_M(t) + v_M(t)] \end{bmatrix} - [D, 1] \begin{bmatrix} x_2(t) \\ x_1(t)u_M(t) + v_M(t) \end{bmatrix} u_T$$

$$(10)$$

This model is in the form of (9) with $P(D) = \text{Row}[D^2,D,1]$, $Q(D) = \text{Row}[D,1]$ and $f(\theta) = \theta = u_T$ - the parameter to be estimated given the data $[x_1,x_2,u_M,v_M]$ on $[t_o,t_1]$. Applying the theory developed in [8], it is then possible to obtain an explicit expression for the estimate $\hat{u}_T$, without estimating the unknown initial conditions $(\dot{x}_1(t_o),\dot{x}_2(t_o))$. Since this theory was developed after the current work of this paper was completed, these expressions will not be written down here and the simulations in Section VI will only incorporate the estimates for (v_T,x_3).

IV. Minimum Energy Control of the Intercept System

Let t_o denote an arbitrary initial time with estimates of the parameters $(v_T,u_T,x_3(t_o))$ assumed given from a previous time interval. Rather than consider $v_M(t)$ and $u_M(t)$ as independent control variables, a proportionality relation

$$v_M(t) = \gamma u_M(t) \tag{11}$$

is postulated with the proportionality parameter γ to be determined from the boundary conditions. The reason for this postulate is based on expediency in obtaining a closed form solution to the following optimal control problem:

$$\text{Minimize } J(u) = \int_{t_o}^{T} u^2(t)dt \tag{12}$$

subject to the intercept condition

$$x_1(T) = x_2(T) = 0 \tag{13}$$

for some finite terminal time $T > t_o$ (a free time formulation), where the notation $u = u_M$ is used for simplicity.

The system (1) can be transformed into a commutative bilinear system by introducing the following auxiliary states:

$$x_4 = \sin x_3, \quad x_5 = \cos x_3, \quad x_6 = 1$$

That is, with these additional states (1) can be written as

$$\dot{x} = Ax + Bu \tag{14}$$

where

$$A = \begin{pmatrix} 0 & 0 & 0 & -v_T & 0 & 0 \\ 0 & 0 & 0 & 0 & v_T & 0 \\ 0 & 0 & 0 & 0 & 0 & u_T \\ 0 & 0 & 0 & 0 & u_T & 0 \\ 0 & 0 & 0 & -u_T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & -\gamma \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad x(t_0) = \begin{pmatrix} x_1(t_0) \\ x_2(t_0) \\ x_3(t_0) \\ \sin x_3(t_0) \\ \cos x_3(t_0) \\ 1 \end{pmatrix}$$

$$\tag{15}$$

It can be readily verified that $AB = BA$ so that (14) is a "commutative" bilinear system. The implication of this fact is that the optimal control solution to (12) and (13), if it exists, is simply a constant function as has been shown in [6], Theorem 3. The existence is contingent upon the terminal condition (13) being a reachable point for the given set of initial conditions. In order to examine this possibility, assume that v_T and u_T are constants so that (14) can be integrated explicitly as follows:

$$x_6(t) = 1 \quad , \quad x_5(t) = \cos x_3(t) \quad , \quad x_4(t) = \sin x_3(t)$$

$$\tag{16}$$

$$x_3(t) = x_3(t_0) + u_T(t-t_0) - \int_{t_0}^{t} u(s)\,ds$$

and

$$\begin{bmatrix} x_1(t) \\ \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t_o)\cos \int_{t_o}^{t} u(s)ds + x_2(t_o)\sin \int_{t_o}^{t} u(s)ds \\ \\ -x_1(t_o)\sin \int_{t_o}^{t} u(s)ds + x_2(t_o)\cos \int_{t_o}^{t} u(s)ds \end{bmatrix}$$

(17)

$$+ v_T \int_{t_o}^{t} \begin{bmatrix} -\sin[x_3(t_o) - \int_{t_o}^{t} uds + u_T(\xi-t_o)] \\ \\ \cos[x_3(t_o) - \int_{t_o}^{t} uds + u_T(\xi-t_o)] \end{bmatrix} d\xi + \gamma \begin{bmatrix} \cos \int_{t_o}^{t} uds - 1 \\ \\ -\sin \int_{t_o}^{t} uds \end{bmatrix}.$$

We shall resolve the terminal constraint problem by considering the intercept angle as a parameter, then incorporate this solution with the minimum energy problem. Consideration should be given to two separate cases in which u_T is zero and non-zero, respectively.

Non-zero Angular Maneuver of the Target $(u_T \neq 0)$

The terminal constraint (13) on x_1 and x_2 requires (for some $T > t_o$):

$$0 = \begin{bmatrix} [x_1(t_o)+\gamma]\cos[u_T(T-t_o)+x_3(t_o)-\beta] + x_2(t_o)\sin[u_T(T-t_o)+x_3(t_o)-\beta]-\gamma \\ -[x_1(t_o)+\gamma]\sin[u_T(T-t_o)+x_3(t_o)-\beta] + x_2(t_o)\cos[u_T(T-t_o)+x_3(t_o)-\beta] \end{bmatrix}$$

$$+ \frac{v_T}{u_T} \begin{bmatrix} \cos \beta - \cos[u_T(T-t_o)-\beta] \\ \sin[u_T(T-t_o)-\beta] + \sin \beta \end{bmatrix} .$$

(18)

where $\beta = x_3(t_o) - \int_{t_o}^{T} u(s)ds + u_T(T-t_o)$ is defined as the intercept angle.

Therefore, the terminal constraint problem has been reduced to solving a pair of transcendental equations, (18), for an appropriate set (γ,β,T). A solution often exists for this case in which the number of unknowns exceeds the number of equations. From Equation (18) we obtain

$$\cos\beta = \cos x_3(t_o) - \frac{u_T}{v_T} x_1(t_o) + \frac{1}{\gamma}\{x_1(t_o)\cos x_3(t_o) + x_2(t_o)\sin x_3(t_o) - \frac{u_T}{2v_T}[x_1^2(t_o) + x_2^2(t_o)]\}.$$

(19)

Given $\{x_1(t_o), x_2(t_o), x_3(t_o), u_T, v_T\}$, appropriate though nonunique values can easily be determined for γ and β such that (19) is satisfied. After γ and β are so determined, the intercept time T can be computed from (18) as follows:

$$T = t_o + \frac{1}{u_T}[2k\pi + \tan^{-1}\frac{F}{G}],$$

(20)

where

$$F = \gamma[\gamma + x_1(t_o)]\sin[\beta - x_3(t_o)] + \gamma\, x_2(t_o)\cos[\beta - x_3(t_o)]$$

$$- \frac{v_T}{u_T}\{\gamma\sin\beta - [\gamma + x_1(t_o)]\sin x_3(t_o) + x_2(t_o)\cos x_3(t_o)\}$$

$$G = \gamma[\gamma + x_1(t_o)]\cos[\beta - x_3(t_o)] - \gamma\, x_2(t_o)\sin[\beta - x_3(t_o)]$$

$$- \frac{v_T}{u_T}\{\gamma\cos\beta + [\gamma + x_1(t_o)]\cos x_3(t_o) + x_2(t_o)\sin x_3(t_o) - \frac{v_T}{u_T}\}$$

$$k = 0, \pm1, \ldots \quad \text{such that } T > t_o.$$

In summary, if a constant but non-zero angular maneuver of the target is assumed, then there exists a triple (γ, β, T) satisfying (19) - (20) which solves the initial and terminal constraint problem (13) - (15) for every $(x_1(t_o), x_2(t_o), x_3(t_o)) \in R^3$. The corresponding proper control action satisfies

$$\int_{t_o}^{T} u(s)ds = x_3(t_o) - \beta + u_T(T-t_o).$$

(21)

Zero Angular Maneuver of the Target $(u_T=0)$

In a similar manner, the terminal constraint becomes

$$0 = \begin{cases} [\gamma+x_1(t_o)]\cos[\beta-x_3(t_o)]-x_2(t_o)\sin[\beta-x_3(t_o)]-v_T(T-t_o)\sin\beta - \gamma \\ \\ [\gamma+x_1(t_o)]\sin[\beta-x_3(t_o)]+x_2(t_o)\cos[\beta-x_3(t_o)]+v_T(T-t_o)\cos\beta \end{cases}$$

where β is as defined in (18). This equation can be reduced to

$$\cos\beta = \cos x_3(t_o) + \frac{1}{\gamma}[x_1(t_o)\cos x_3(t_o) + x_2(t_o)\sin x_3(t_o)]. \qquad (22)$$

Finally, the intercept time T is determined by

$$T = t_o + \frac{1}{v_T}\{[\gamma+x_1(t_o)]\sin x_3(t_o) - x_2(t_o)\cos x_3(t_o) - \gamma\sin\beta\}. \qquad (23)$$

A feasible T requires $T > t_o$, i.e., the term in the bracket must be positive. It can be shown (see Section 3.2 in [9]) that this is true only for those initial conditions outside the region E defined by

$$E = \{(0,y,z) \in R^3: \text{ either } y > 0, z = (2k+1)\pi, \text{ or } y < 0, z = 2k\pi; k = 0,\pm1,...\}$$

Therefore, if a zero angular maneuver of the target is assumed, then there exists a triple (γ,β,T) which solves the initial and terminal constraint problem (13) - (15) for every $(x_1(t_o),x_2(t_o),x_3(t_o)) \in R^3 \setminus E$.[†]

It should be noticed that in the previous analyses the control function $u(t)$ has been eliminated for simplicity of computation. However, an admissible control which steers the missile to the target at T is associated with the triple (γ,β,T) through Equation (21). Thus the set U_c of admissible controls is specified by:

$$U_c = \{u \in L^2([t_o,T],R): \int_{t_o}^{T} u(s)ds = x_3(t_o)-\beta+u_T(T-t_o)\} \qquad (24)$$

where β and T are determined by Equations (19) - (20), or (22) - (23).

[†] $A \setminus B$ denotes complement of B in A.

With the set U_c of admissible controls furnished as in (24), we are ready to state the solution to the minimum energy problem (12). The following proposition is a direct consequence of Theorem 3 in [6].

Proposition 1:

Given the system (14) - (15), there exists an optimal control $u^* \varepsilon U_c$ which minimizes the cost (12) subject to the constraint (13) for each appropriate set of initial conditions $(x_1(t_o), x_2(t_o), x_3(t_o), u_T, v_T)$. This control is given by

$$u^*(t) = u_T + \frac{x_3(t_o) - \beta}{T - t_o} \qquad (25)$$

where T and β are given in Equations (19) - (20), or (22) - (23).

V. Singularly Perturbed Problem

A more complex system model is considered in this section in which the missile turn rate is taken as the output of a first-order lag. This takes into account the practical situation in which the missile turn rate is furnished by a DC-motor having first-order actuator dynamics. That is,

$$\varepsilon \, \dot{u}(t) = -u(t) + u_o \qquad \varepsilon > 0 \qquad (26)$$

where u_o is the real input. In the limiting case where ε approaches zero, this consideration is generally known as a 'singular perturbation' problem [10].

The cost functional in this case becomes

$$J(u_o) = \frac{1}{2} \int_{t_o}^{T} u_o^2(s) ds \qquad (27)$$

subject to the constraint

$$x_1(T) = x_2(T) = 0 \quad \text{for some } T > t_o. \qquad (28)$$

By defining $z = u$, the system equations can be expressed by:

$$\dot{x} = Ax + Bxz$$

$$\varepsilon > 0 \qquad (29)$$

$$\varepsilon \dot{z} = -z + u_o$$

where A and B are the same as defined in (15).

Because it was shown in the last section that the terminal constraint problem has a solution when the control action satisfies (21), or in terms of the new state z

$$\int_{t_o}^{T} z(s)ds = x_3(t_o) - \beta + u_T(T-t_o),$$

the terminal constraint problem (28) also has a solution provided:

$$\frac{1}{\varepsilon}\int_{t_o}^{T} \int_{t_o}^{t} u_o(s)e^{-\frac{t-s}{\varepsilon}} ds \, dt = x_3(t_o) - \beta + u_T(T-t_o) + \varepsilon z(t_o) \left[e^{-\frac{T-t_o}{\varepsilon}} - 1 \right] . \qquad (30)$$

Thus the set U_c of admissible controls for the problem (27) - (29) is the collection of inputs u_o satisfying (30).

The existence of an optimal control $u_o^* \in U_c$ which minimizes the cost (27) can be easily established and by the Maximum Principle this optimal solution satisfies

$$u_o^*(t) = q(t) \qquad (31)$$

and

$$\dot{p}(t) = -\partial H/\partial x = -(A'+B'z)p$$

$$, \quad \varepsilon q(T) = 0. \qquad (32)$$

$$\varepsilon \dot{q}(t) = -\partial H/\partial z - -x'B'p+q$$

In the latter expression, (p,q) are costates corresponding to (x,z) and prime denotes the matrix transpose operation. It can be easily checked that $\frac{d}{dt}(x'B'p) = 0$; hence $x'(t)B'p(t) = k$, a constant to be determined by the boundary conditions.

Substituting k into (32), q can be solved:

$$q(t,\epsilon) = -k\left[e^{-\frac{t-T}{\epsilon}} - 1\right] = u_o^*(t). \tag{33}$$

Then from Equations (30) - (31), k is given by:

$$k = \frac{1}{L}\left[x_3(t_o) - \beta + u_T(T-t_o) + \epsilon z(t_o)(e^{-\frac{T-t_o}{\epsilon}} - 1)\right] \tag{34}$$

where

$$L \approx \epsilon\left[-1.5 + \frac{1}{2}e^{\frac{t_o-T}{\epsilon}} + \frac{1}{2}e^{\frac{2(t_o-T)}{\epsilon}}\right] + T - t_o.$$

These results are summarized in the next proposition.

Proposition 2:

Given the system (31), there exists an optimal control $u_o^* \epsilon U_c$ which minimizes the cost (27) subject to the constraint (28) for each appropriate set of initial conditions. This control is given by (33) and (34) where β and T are specified in Proposition 1.

It should be noticed that the singular perturbation comes into the problem (29) as ϵ appraches zero. This is clearly seen from the expression of the optimal control (33) - (34), i.e.

$$\lim_{\epsilon \to 0} k(\epsilon) = \frac{1}{T-t_o} [x_3(t_o) - \beta + u_T(T-t_o)]$$

$$\lim_{\epsilon \to 0} u_o^*(t,\epsilon) = -\lim_{\epsilon \to 0} k(\epsilon) \left[e^{\frac{t-T}{\epsilon}} - 1 \right]$$

$$= u_T + \frac{x_3(t_o) - \beta}{T-t_o} \qquad t_o \leq t \leq T,$$

which is exactly the same as that in (25). Therefore, for ϵ sufficiently small the solution to the optimal control problem (27) associated with a fourth-order system (29) can be approximated arbitrarily closely by the solution to the problem (12) associated with the third-order system (1).

In fact, not only the reduction of system order is shown here, but also an explicit solution to the optimization problem of the quadratic system[†] is derived. This provides a great deal of potential to implement such a control law in practice because the first order actuator dynamics have been included. Actually, this result can be generalized to include any higher order actuator dynamics as long as the constancy of the control area is sustained.

VI. Simulation Results

The least squares estimation scheme of Section III is combined with the optimal control law of Sections IV and V to form a step-by-step feedback control of the missile intercept system for selected initial conditions and target turn rates. The initial target turn rates are assumed to be 0.0 rad/sec and 0.15 rad/sec while the target line speed is 1000ft/sec. In addition, two sets of sinusoidal fluctuations were imposed upon the target turn rate and line speed as listed in Table 1 to test the sensitivity of the system.

[†]Note that (29) is no longer a bilinear system as defined in Section IV; instead, it is sometimes referred to as a quadratic system.

TABLE 1.

Actual Target Turn Rates and Line Speeds

Figure No.	u_T (rad/sec)	v_T (ft/sec)
2	0.15	1000
3	0.00	1000
4	$-0.001+0.001(\cos 0.2t+\sin 0.2t)$	1000
5	0.00	1000
6	0.00	$980+20(\cos 0.2t+\sin 0.2t)$

A time constant $\varepsilon = 0.5$ sec was assumed for the first- order actuator dynamics. During the first estimation interval of duration 0.5 sec., the control effort was assumed to be $u(t) = 0.06e^{-2t}$ rad/sec. Thereafter, the estimation intervals were determined by Equations (20) or (23) (depending on whether u_T is zero or nonzero) and consisted of a single interval when the estimation over $0 \leqslant t \leqslant 0.5$ was sufficiently accurate, as in Figs. 2, 3 and 5 where one interval of control led to an interception. Additional estimation and control intervals were needed for the perturbed target turn rates and line speed cases shown in Figs. 4 and 6. It is clear that in the case where the target turn rate and line speed are constant (Figs. 2, 3 and 5), the estimation of the target speed and the initial relative heading is exact and an ideal intercept is achieved in one interval as expected. For the case in which a fluctuation of either the target turn rate or its line speed is not known in advance (Figs. 4 and 6), the step-by-step control action is taken to offset the estimation error, which prolongs the expected intercept interval. Nevertheless, reasonable convergence occurs after a few loops of estimation in these particular cases.

Unknown fluctuations on the target turn rate of higher amplitude, e.g. $-0.005+0.005(\cos 0.2t+\sin 0.2t)$ were also considered, as well as a time-varying

sawtooth target line speed, in order to test the effectiveness of the estimation scheme. Simulation results are relatively poor in these cases even though further estimation steps are called upon in trying to reduce the estimation error. This indicates the high sensitivity of the estimation scheme to deviations from the assumed constant values for the unknown parameters. It is suggested that a recursive estimator, which utilizes the first estimation data to initiate a secondary minimum variance estimation on either the target turn rate or line speed, may be worthy of consideration for future analyses.

VII. Concluding Remarks

The control law proposed here for a missile intercept problem is admittedly ad hoc. Separating the estimation and control problems, then combining their solutions in a step-by-step fashion, does not in any way constitute an optimal solution for the over-all problem. Moreover, the assumption that the pursuer possesses thrust modulation in addition to the usual thrust vectoring precludes the application to many present day systems. Nevertheless, this assumption does allow for a simple closed form solution to the minimum control energy-terminal constraint problem and, it is believed, has enough elements of practicality to make it potentially attractive for future systems which may possess thrust modulation capabilities. The simulation results indicate good accuracy for an intercept when the estimation errors are small. At the same time, the simulations for various amplitudes of the fluctuations in the target speed and turn rate reflect the possible necessity for a higher order, or more sophisticated, estimation scheme to alleviate such fluctuation effects on terminal accuracy.

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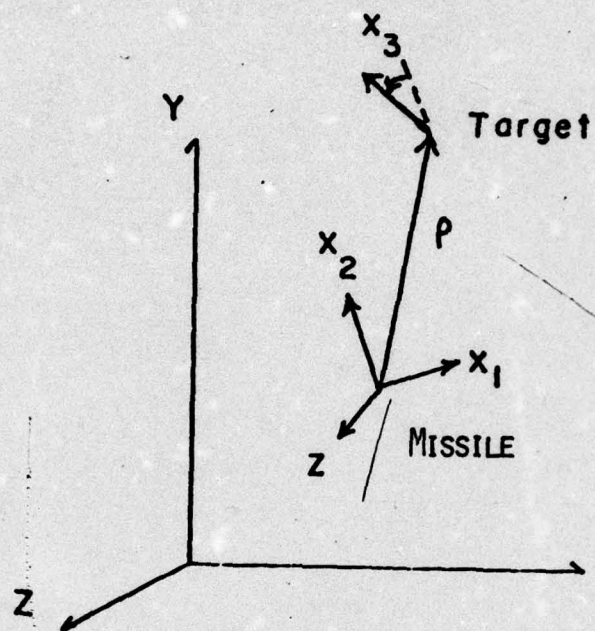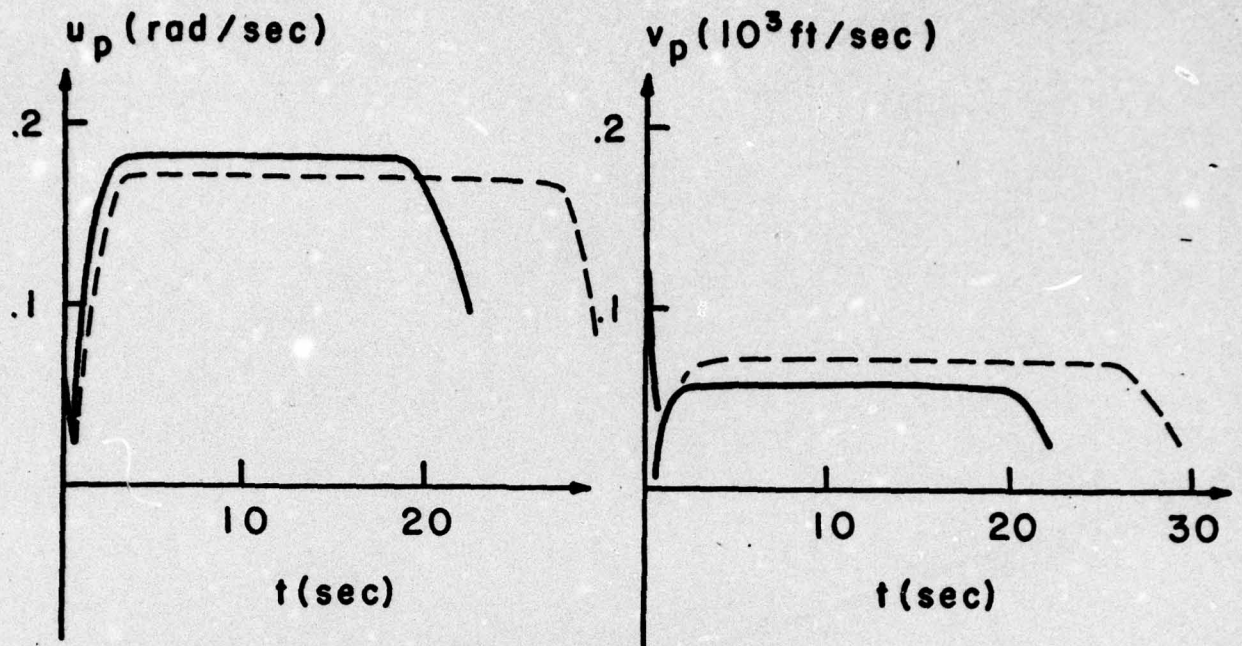
Fig. 1 Geometry for a Two-Dimensional Intercept System

u_p (rad/sec)

$v_p (10^3 ft/sec)$

.2

.2

.1

.1

10   20

10   20   30

t (sec)

t (sec)

Trajs.	Initial Data		End Pt. Data	
	$\|X(0)\|$	$\beta(0)$	$\|X(T)\|$	$\beta(T)$
——	10000	57.29°	0	28.64°
- - - -	8060	57.29°	0	28.64°

$x_2 (10^3 ft)$

10
8
6
4
2

-4 -2   2   4   6   8

$x_1 (10^3 ft)$
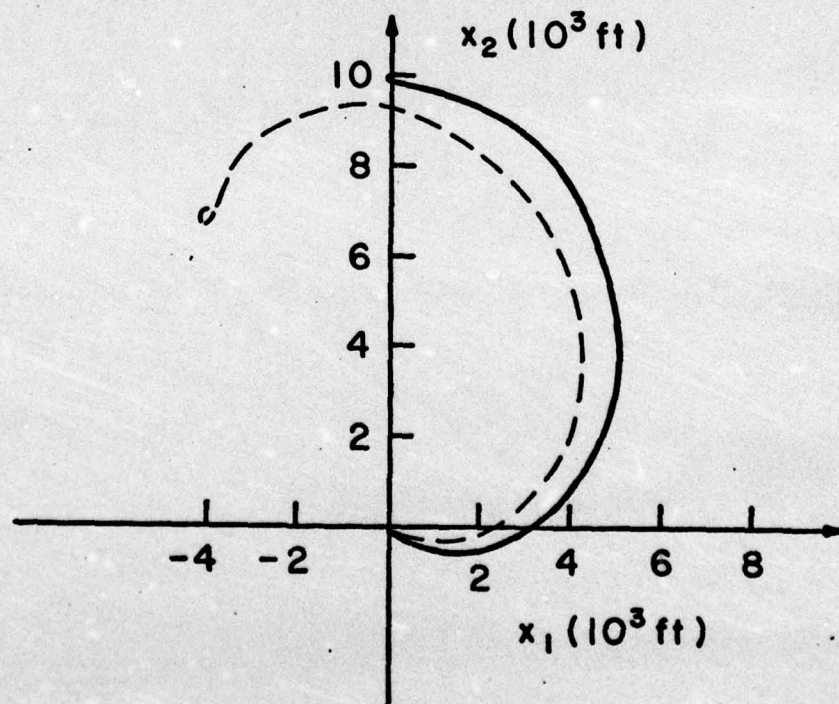
Figure 2. Missile Turn Rate, Speed and Relative Trajectories
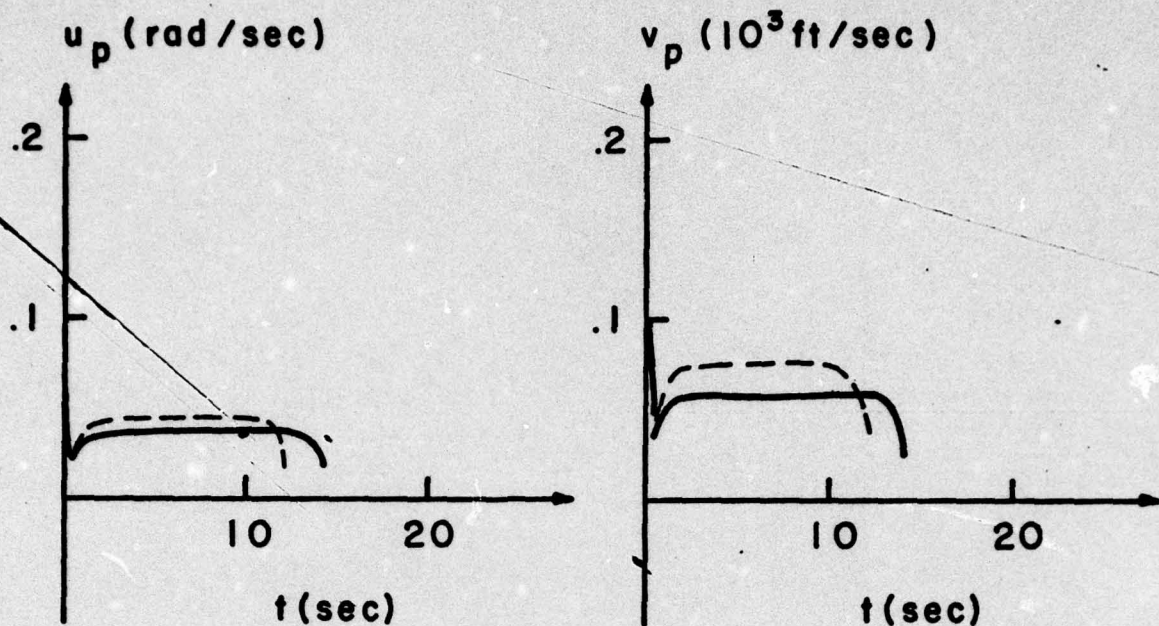
Trajs.	Initial Data		End Pt. Data	
	‖X(O)‖	β(O)	‖X(T)‖	β(T)
———	10000	57.29°	0	28.64°
- - - -	8250	57.29°	0	28.64°
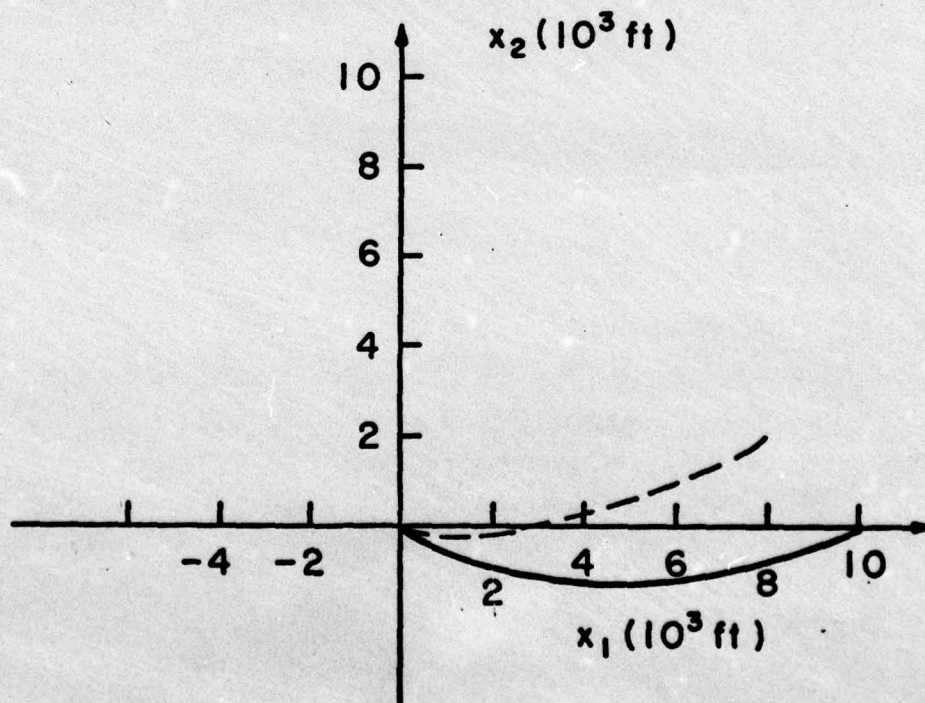
Figure 3. Missile Turn Rate, Speed and Relative Trajectories

u_p (rad/sec)

.2

.1

10   20

t (sec)

v_p (10^3 ft/sec)

.2

.1

10   20

t (sec)
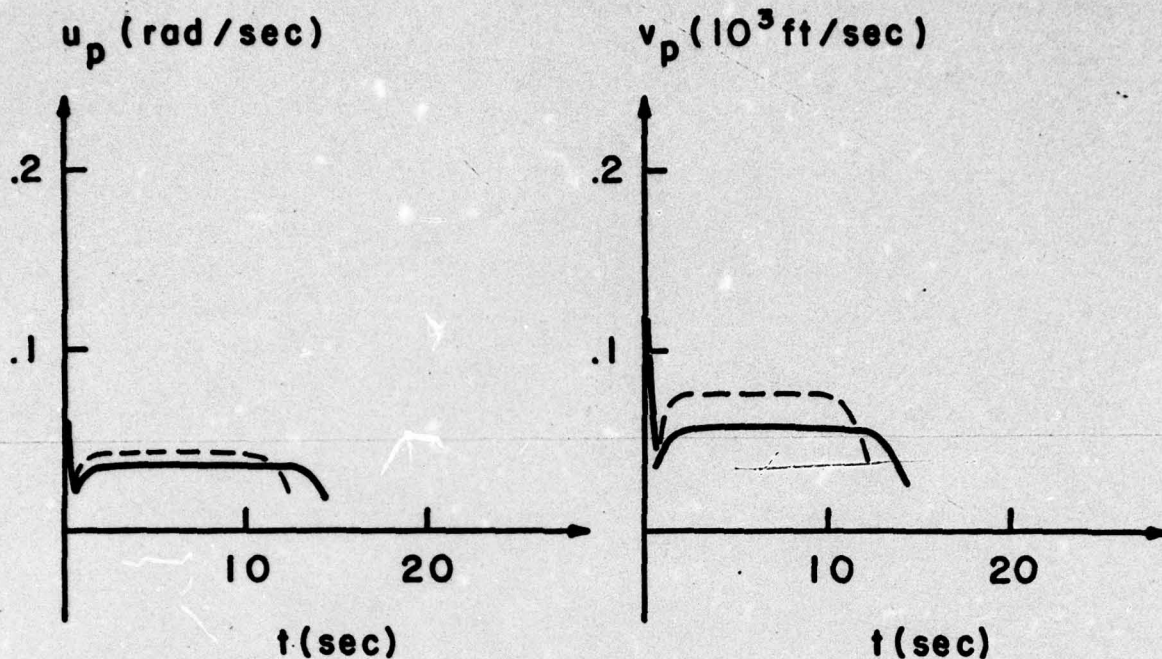
Trajs.	Initial Data		End Pt. Data	
	$\|X(0)\|$	$\beta(0)$	$\|X(T)\|$	$\beta(T)$
——	10000	57.29°	11.5	28.5°
-----	8250	57.29°	14.5	28.6°

x_2 (10^3 ft)

10

8

6

4

2

-4   -2   2   4   6   8   10

x_1 (10^3 ft)
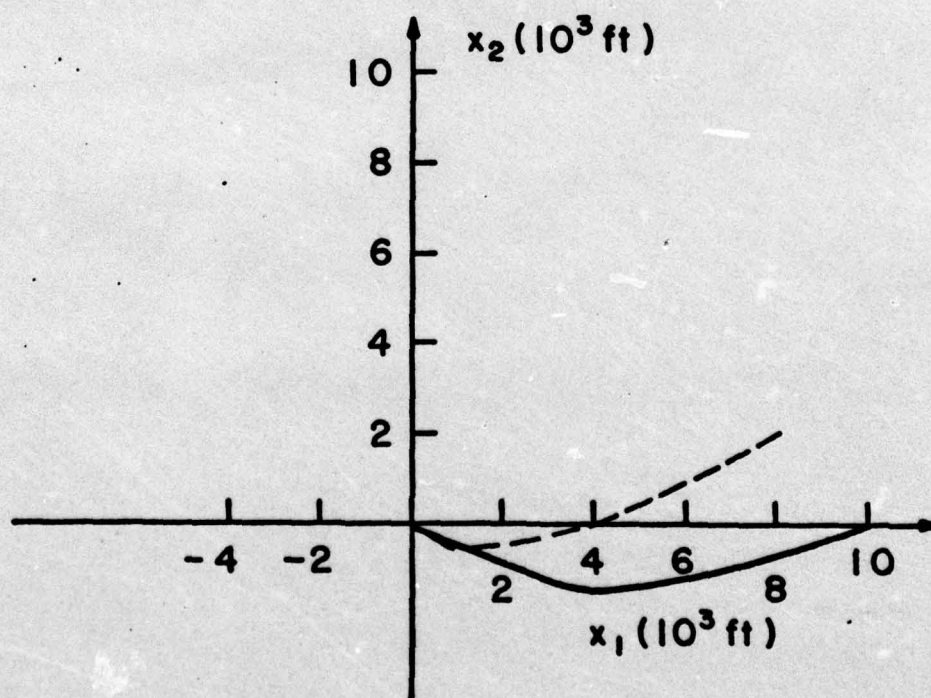
Figure 4. Missile Turn Rate, Speed and Relative Trajectories
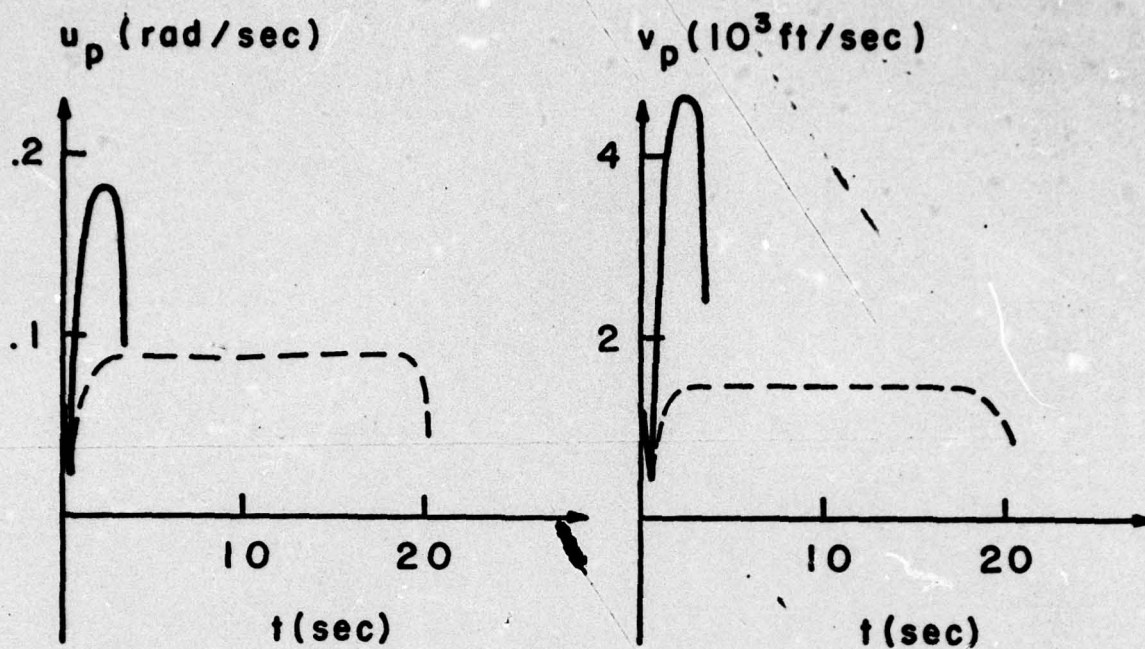
Trajs.	Initial Data		End Pt. Data	
	$\|X(0)\|$	$\beta(0)$	$\|X(T)\|$	$\beta(T)$
——	10000	57.29°	6.56	28.63°
- - - -	8060	57.29°	0	-41.88°
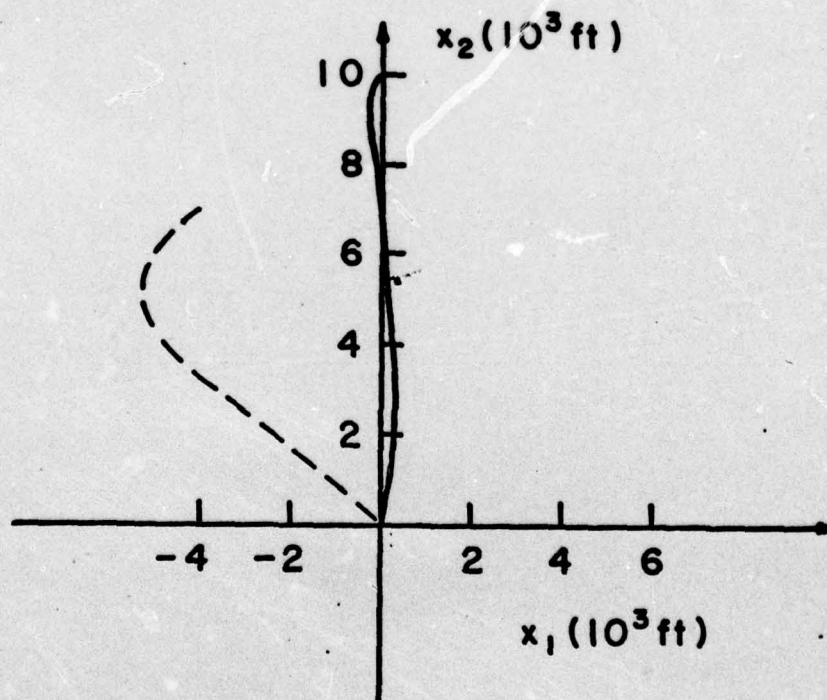
Figure 5. Missile Turn Rate, Speed and Relative Trajectories
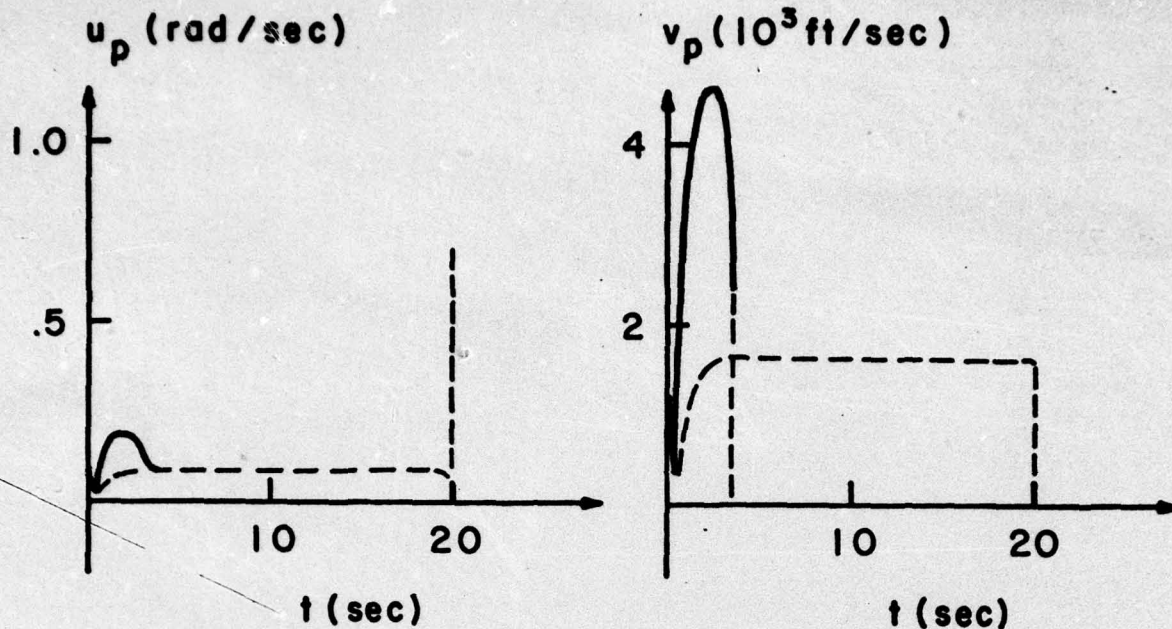
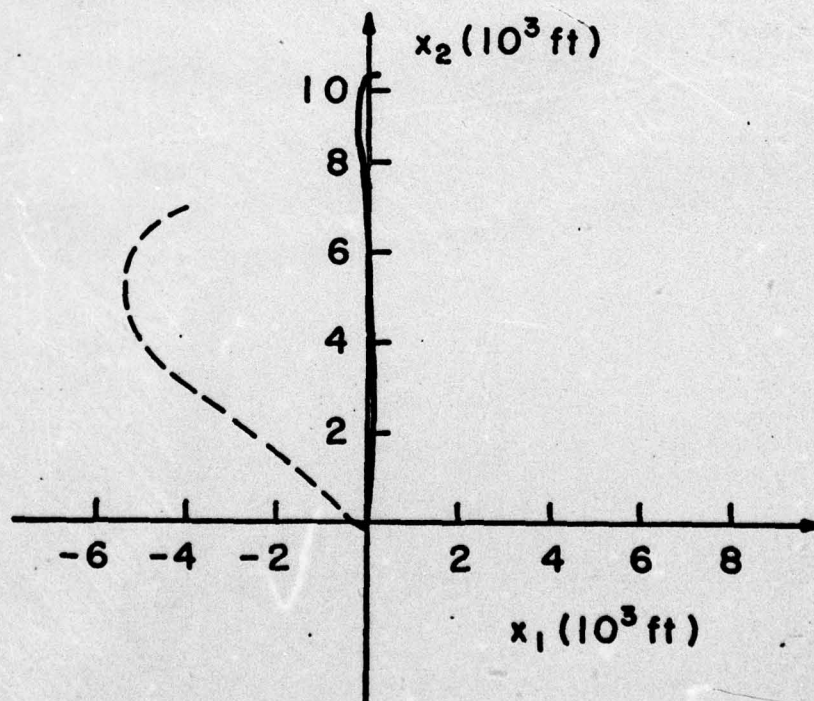| Trajs.	Initial Data		End Pt. Data	
	‖X(0)‖	β(0)	‖X(T)‖	β(T)
———	10000	57.29°	10.32	28.60°
- - - -	8060	57.29°	0.03	-54.03°

Figure 6. Missile Turn Rate, Speed and Relative Trajectories

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A feedback control law is derived for a two dimensional missile intercept system which combines in a step-by-step manner the closed form solutions to a least squares estimation of the target turn rate, line speed and heading with a minimum control energy problem subject to a terminal intercept constraint.

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20. ABSTRACT (Continued)

The major assumptions are that the pursuer possesses thrust modulation capabilities, in addition to thrust vectoring, and that continuous measurements of the range coordinates are available for estimation purposes. Simulation results are included which indicate that good terminal accuracy is achieved when the estimation errors are small and illustrate the limitations on accuracy when fluctuations occur in the target speed and turn rate.